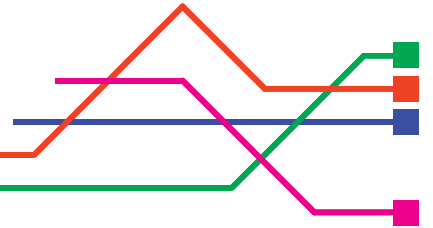

SLOT DESIGNER

Tools for professional mathematicians



Elements Of Slot Design

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<http://slotdesigner.com/>

CHAPTER 1 INTRODUCTION

This document is intended as an introduction to Slot game mathematics and design. In time it should expand to provide a comprehensive tutorial and reference to all aspects of Slot design.

The fundamentals of Slot mathematics are introduced so that games can be designed from the ground up in Microsoft Excel, code, or using Slot Designer. It assumes a basic understanding of probability and Slot machines.

Unfortunately there is no standardized method of performing these calculations, designing spread sheets, or even terminology. Different countries and companies for example refer to symbols as icons, and reel strips as reel bands. This document therefore uses the terminology defined by Slot Designer, which is the commercial product of the author.

While the intent is to produce a very much larger book, practical constraints dictate this will take some time. In the meantime feedback would be appreciated as to the content and future direction.

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CHAPTER 2 THE BASICS

We'll start by considering a very simple slot game with 3 reels, each of 10 symbols, 1 pay line, and 1 pay rule, and a cost to play each game of 1 credit. Each reel contains 10 symbols in total, with 1 A and 9 X symbols,

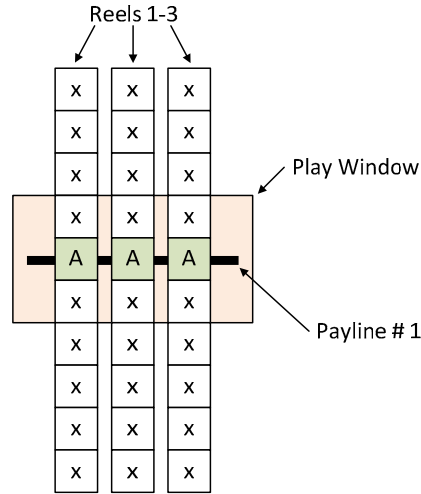


Figure 1. Simple 3x3 Game

The game's single pay rule awards 1000 credits when we line up 3 A symbols across the pay line. When spinning the reels we can see the probability of stopping with an A on the payline for each reel is 1/10, hence the probability of getting 3 A's across the 3 reels is 1/1000.

$$p(A, A, A) = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000} \quad \text{Equation 1}$$

When we win the rule awards 1000 credits, so the average win per game is 1 credit

$$\text{average win} = \text{prize} \times \text{probability of prize} \quad \text{Equation 2}$$

$$\text{average win} = \text{prize} \times p(a, a, a) = 1000 \times \frac{1}{1000} = 1 \text{ credit}$$

When we play the game (spin the reels) we pay 1 credit, so the average cost to play each game (the bet) is simply 1 credit.

The RTP (return to player) of a game is defined as the average win as a percentage of the bet, which for this game is 100%.

$$RTP = \frac{\text{average win}}{\text{average bet}} \times 100\% \quad \text{Equation 3}$$

$$RTP = \frac{1}{1} \times 100\% = 100\% \quad \text{Equation 4}$$

In other words, over the long term as we play this game, we will win 100% of what we bet even though we win only 1 once in 1000 games. When playing a game with an RTP over 100% the player wins money over the long term, and below 100% the player loses money.

If we were to change the prize value from 1000 credits to 500 credits, then the average win (Equation 2) is 0.5 credits, and the RTP (Equation 3) is 50%.

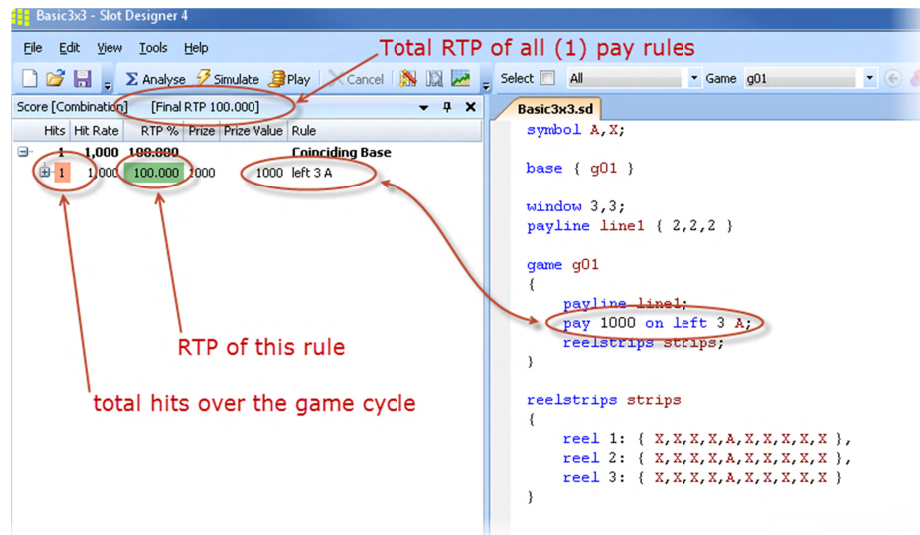
If we were to add a second A symbol on the first reel only, we would double the probability of A on that reel, hence doubling the probability of 3 A on the payline occurring. With the original 1000 credit prize, the average win is then 2 credits (Equation 2), and the RTP 200% (Equation 3). We now win once in every 500 games.

$$\begin{aligned}
 p(A, A, A) &= \frac{2}{10} \times \frac{1}{10} \times \frac{1}{10} \\
 &= \frac{2}{1000} \\
 &= \frac{1}{500}
 \end{aligned}$$

Combining a second A on the first reel with the prize value of 500 credits, we're doubling the probability and halving the prize, hence the RTP remains at 100%. With 2 A on each of the 3 reels, we would have 8 times the probability, resulting in an RTP of 400% (with the 500 credit prize).

The winning pattern of 3 A symbols is called a "combination", and the complete list of combinations comprises the games pay table. We can write this as {A,A,A}.

In Slot Designer this game we can implement the game with the script shown on the right, and the basic statistical report for the pay rules on the left.



COUNTING HITS

Rather than perform all the calculations as probabilities, we usually count the number of winning combinations over the game cycle and divide by the cycle at the end. The difference is subtle and end result is the same, but it's easier to work with. We can rewrite Equation 1 using hits instead of probability

$$\begin{aligned}
 p(A, A, A) &= \frac{\text{hits}}{\text{cycle}} \\
 &= \frac{1 \times 1 \times 1}{10 \times 10 \times 10} \\
 &= \frac{1}{1000}
 \end{aligned}
 \tag{Equation 5}$$

HIT RATE AND HIT FREQUENCY

The hits rate tells us how many games we have to play on average to win a particular prize, while the hit frequency tells us what percentage of games win the prize. This particularly useful when looking at how often the player will trigger a feature game.

The hit rate can be calculated several different ways

$$\text{hitrate} = \frac{1}{\text{probability}} \quad \text{Equation 6}$$

$$\text{hitrate} = \frac{\text{cycle}}{\text{hits}} \quad \text{Equation 7}$$

And the hit frequency

$$\text{hit frequency} = \frac{100\%}{\text{hit rate}} \quad \text{Equation 8}$$

In the game of Equation 1 the hit rate is 1000, or 1000 games between hits.

$$\begin{aligned} \text{hit rate}(A, A, A) &= \frac{\text{cycle}}{\text{hits}} \\ &= \frac{1000}{1} \\ &= 1000 \end{aligned} \quad \text{Equation 9}$$

And the hit frequency is 0.1%

$$\begin{aligned} \text{hit frequency}(A, A, A) &= \frac{100\%}{1000} \\ &= 0.1\% \end{aligned} \quad \text{Equation 10}$$

VOLATILITY

If we have a game that wins 1 credit for every play, the average win is then 1 credit, and with an average bet of 1 credit this gives us an RTP of 100% (Equation 3). In the previous section we also calculated a RTP of 100% for a game that wins 1000 credits once in every 1000 games, and zero every other play.

These two games have the same RTP, yet would be fundamentally different to play. The key difference between these two games lies in their volatility. The game that wins every time has a very low volatility, and the other a very high volatility. Indeed, their volatility is extreme that neither would be a practical design.

We can also see that as the chance of winning increases the prize won must correspondingly decrease to achieve the same RTP.

SIMPLE PAY RULES

We can add a second pay rule to the game in Figure 1, which wins 100 credits when we get A on the left 2 reels, but not on the 3rd reel. This is commonly referred to as a left to right combination, and we can write it as the combinations {A,A,#A}, where # means "not".

Counting the hits for this rule we have 1 on reels 1 and 2 as before, but on reel 3 there are 9 hits where A does not occur. We don't count 10 hits for 'any' symbol as matching the combination on reel 3, as another A on reel 3 would match the left 3 A pay rule instead. Left 2 A and left 3 A are mutually exclusive.

The number of hits is then

$$\begin{aligned} \text{hits}(\text{left } 2 \text{ } A) &= \text{hits}(A, A, \# A) \\ &= 1 \times 1 \times (10 - 1) \\ &= 9 \end{aligned} \quad \text{Equation 11}$$

The average win due to this pay rule is then 0.9 credits (using Equation 2 and Equation 5)

$$\begin{aligned}
 \text{average win} &= \text{prize} \times \frac{\text{hits}(\text{left } 2 \text{ a})}{\text{cycle}} & \text{Equation 12} \\
 &= 100 \times \frac{9}{1000} \\
 &= 0.9 \text{ credits}
 \end{aligned}$$

Hence, using Equation 3, the RTP contributed to the game by this pay rule is 90%. As the left 3 A rule contributes 100%, the total RTP of the game is simply the sum of the individual contributions, or 190%.

Instead of a left to right pay rule we could add instead an 'any' pay rule, such that any 2 A pays 100 credits. This pay rule comprises 3 combinations, {A,A,#A}, {A,#A,A}, and {#A,A,A}, each of which has exactly 2 A symbols, and are mutually exclusive. There are 27 hits on any 3 A

$$\begin{aligned}
 \text{hits}(\text{any } 2 \text{ A}) &= \text{hits}(A, A, \# A) + \text{hits}(A, \# A, A) + \text{hits}(\# A, A, A) & \text{Equation 13} \\
 &= 1 \times 1 \times (10 - 1) + 1 \times (10 - 1) \times 1 + (10 - 1) \times 1 \times 1 \\
 &= 9 + 9 + 9 \\
 &= 27
 \end{aligned}$$

And the RTP for this pay rule is therefore 270%, and the total RTP for the game is 370%.

$$\begin{aligned}
 \text{RTP}(\text{any } 2 \text{ A}) &= \text{prize} \times \frac{\text{hits}(\text{any } 2 \text{ A})}{\text{cycle} \cdot \text{bet}} & \text{Equation 14} \\
 &= 100 \times \frac{27}{1000} \times 100\% \\
 &= 270\%
 \end{aligned}$$

Adding another 'any' rule to pay 10 credits on "any 1 A", we have combinations {A,#A,#A}, {#A,A,#A}, and {#A,#A,A}. There are 243 hits due to this pay rule.

$$\begin{aligned}
 \text{hits}(\text{any } 2 \text{ A}) &= \text{hits}(A, \# A, \# A) + \text{hits}(\# A, A, \# A) + \text{hits}(\# A, \# A, A) & \text{Equation 15} \\
 &= 1 \times (10 - 1) \times (10 - 1) + (10 - 1) \times 1 \times (10 - 1) + (10 - 1) \times (10 - 1) \times 1 \\
 &= 81 + 81 + 81 \\
 &= 243
 \end{aligned}$$

The RTP for this rule is therefore 243%.

$$\begin{aligned}
 \text{RTP}(\text{any } 1 \text{ A}) &= \text{prize} \times \frac{\text{hits}(\text{any } 1 \text{ A})}{\text{cycle} \cdot \text{bet}} & \text{Equation 16} \\
 &= 10 \times \frac{243}{1000} \times 100\% \\
 &= 243\%
 \end{aligned}$$

The RTP for the game is the sum of the RTP's of each of its pay rules, which for the 3 "any" rules is 613%

$$100\% + 270\% + 243\% = 613\%.$$

SCATTER

Normal, or non-scattered symbols, only pay (hit) when they are on the payline. A scattered symbol hits when it is visible in the play window anywhere on the reel, i.e. on, above or below the payline.

As the total hits over the cycle remain the same the RTP is unchanged. However the prizes awarded by the game are different. When a win is due to the 2 stacked symbols the number of hits doubles, hence the prize for that win is also doubled. While the RTP is the same, the volatility is slightly changed.

Previously we counted 4 hits of #S with 2 separated scatter symbols on the reel, but now there are 6 hits with no compensating change in the number of hits at each – these remain zero. Hence we can see that the reelstrip layout can change the RTP of games that have scatter symbols. Normally scatter symbols are separated on the reels, so that multiple scatter symbols cannot be seen on the same reel at once.

When designing games, especially using Excel, the calculations are typically performed assuming scatters are spread out on the reels so that this effect does not occur, and when the actual game is built the symbols are spread out to match this assumption. In the event that the scatters are not properly separated an error will result.

In Slot Designer a symbol can be globally scattered with the scatter command.

```
scatter A;
```

We can also scatter it only where it is used in a combination

```
pay 100 on left 3 A.s;
```

WILD SYMBOLS

A wild or substitute symbol is one that can be used in place of other symbols in the pay table combinations.

Consider the game of Figure 1 (page 4), where one of the X symbols is replaced by the wild symbol 'W' on reel 3.

x	x	x
x	x	x
x	x	A
x	x	x
A	A	W
x	x	x
x	x	x
x	x	x
x	x	x
x	x	x

For the combination {A,A,A} we have 1 hit on reels 1 and 2, but on reel 3 both the A and W will hit for this combination, hence we now have 2 hits where before we had only one.

$$\begin{aligned} hits(A, A, A) &= 1 \times 1 \times 2 \\ &= 2 \end{aligned}$$

An alternate notation to represent this combination, embedding the wild, is

{A,A,{A,W}}

This indicates that A or W is counted on the 3rd reel of the payline.

This is equivalent to adding a second A to reel 3, however if we had other symbol and combinations, the wild symbol could substitute for other symbols than just A.

One well known variation on substitution is to double the prize when a wild is used to match the combination, so that {A,A,W} would pay double the prize of {A,A,A}. This simply requires counting the hits for the various combinations, and determining the RTP from the appropriate prize values.

The RTP of this game where 3 × A pays 100 credits, W is wild and doubles the prize, and the bet is 1 credit, is 30%.

$$hits(A, A, A) = 1 \times 1 \times 1 = 1$$

Equation 17

$$hits(A, A, W) = 1 \times 1 \times 1 = 1$$

$$RTP(A, A, A) = \frac{hits \times prize}{cycle \times bet} \times 100\% = \frac{1 \times 100}{1000 \times 1} = 10\%$$

$$RTP(A, A, W) = \frac{1 \times 200}{1000 \times 1} = 20\%$$

$$\begin{aligned} Total \text{ RTP} &= 10\% + 20\% \\ &= 30\% \end{aligned}$$

In Slot Designer substitution can be implemented in several ways. We can include the wild symbol 'W' in the pay rules

```
pay 100 on left 3 { A, W };
```

Alternately we can use the substitute command to automatically modify all the combinations to include the wild symbol.

```
substitute W = all;  
pay 100 on left 3 A;
```

The wild symbol does not usually apply to the scattered symbol 'S'.

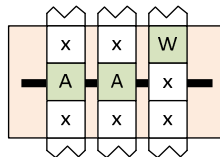
```
substitute W = all except S;
```

We can automatically double the prize value when the wild symbol is used.

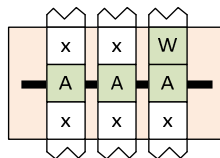
```
substitute W = all except S pay = pay * 2;
```

EXPANDING WILD

A popular variation on substitution is to scatter the wild symbol, so that if the wild is anywhere on the reel it substitutes into the payline combination. Normally the wild symbol substitutes only when it is on the payline. In this example the combination {A,A,A} hits due to the off-payline scattered wild symbol W.



As with other types of scattered symbol this can result in multiple hits on the reel when the combination symbol is on the payline (1 hit) and the wild symbol is above or below the payline (1 more hit).



Given that an expanding wild usually displays an animation that expands to cover the entire reel and payline combination symbol cannot be seen, the hit from the payline symbol is usually not counted – i.e. we count 1 hit, not 2. We can then use the RTP contribution saved in another part of the game where it will be visible to the player.

In Slot Designer expanding wild (where the wild symbol W substitutes for A and is scattered) can be implemented as

```
pay 100 on left 3 { A, W.s };
```

When we want to reduce multiple hits to a single hit we use the merge attribute

```
pay 100 on left 3 { A, W.s }.merge;
```

PRIORITISATION

Consider a game where the wild symbol W substitutes for symbols A and B, and the reels stop with symbols {W, W, A} on the payline. This matches both left 3 A and left 2 B pay rules (combinations {A,A,A} and {B,B,#B} respectively).

While we could pay both, it is more common to pay on the highest win only, hence the common rule on slot games of "Highest win pays on each payline". This is also referred to as prioritization, as we order the priority of the pay rules, and determine which single rule to pay.

A	x	x
W	x	x
B	x	x
A	x	B
W	W	A
B	x	A
W	x	x
A	x	x
W	x	x
A	x	x

Left 3 A pays 200
Left 2 B pays 100

The hits without prioritization, including wild, are

$$\begin{aligned} hits(A, A, A) &= (4 + 4) \times 1 \times 2 \\ &= 16 \end{aligned} \quad \text{Equation 18}$$

$$\begin{aligned} hits(B, B, \#B) &= (2 + 4) \times 1 \times (10 - 1) \\ &= 54 \end{aligned} \quad \text{Equation 19}$$

The RTP for the non-prioritized game is then 860%

$$\begin{aligned} rtp &= \frac{16 * 200 + 54 * 100}{1} \times 100\% \\ &= 860\% \end{aligned} \quad \text{Equation 20}$$

Consider the winning combination {W,W,A}. When all wins pay this would hit both left 2 B and left 3 A. When only the highest win pays then we need to decide to pay either the 2 B or 3 A, and choose 3 A as it's the highest (otherwise the player would be quite upset). We need adjust, or discount, the hits for left 2 B to exclude those hits that pay as left 3 A instead.

The combinations where we pay left 3 A instead of left 3 B are those where stop with {W,W,A} on the payline, for which there are 8 hits

$$\begin{aligned} hits(W, W, A) &= 4 \times 1 \times 2 \\ &= 8 \end{aligned} \quad \text{Equation 21}$$

The number of hits on left 2 B is then $54 - 8 = 46$ hits.

The RTP for the game is 780%, a considerable drop from the non-prioritized 860%.

$$\begin{aligned} rtp &= \frac{16 * 200 + 46 * 100}{1} \times 100\% \\ &= 780\% \end{aligned} \quad \text{Equation 22}$$

Where pay rules have the same prize value the priority is chosen by the game designer.

In a real game using Excel the discounting gets quite involved.

Prioritization in slot designer is implemented as

```
prioritise highest
{
  pay 200 on left 3 A;
  pay 100 on left 2 B;
}
```

MULTILINE

Most games have more than one payline, and some have 100 or more. There is no standard layout for paylines, apart perhaps from the first 3.



The number of paylines and their layout is usually of little concern mathematically, as the RTP tends to be independent of the number of paylines in the game; in Excel the RTP is usually calculated for a single payline only. The hits and therefore win increases linearly with additional paylines, hence the bet is similarly increased to keep the RTP constant.

As hits for scatter combinations are independent of paylines their prizes are multiplied by the bet to keep the RTP constant.

The maximum number of paylines in a game is the product of the individual window heights for each reel, and in games where it is used is sometimes described as “ways to win”. In a 5 reel 3 row games this is 3^5 , or 243 lines or ways to win. Similarly for a 5 by 4 game there’s a maximum of 4^5 or 1024 paylines. A game with a window of 3, 4, 5, 4, 3 symbols visible on reels 1 to 5 respectively would have a maximum of $3 \times 4 \times 5 \times 4 \times 3 = 720$ paylines.

The 5 paylines shown above were created by the following Slot Designer code

```
payline line01 { 2,2,2,2,2 } // define the lines
payline line02 { 1,1,1,1,1 }
payline line03 { 3,3,3,3,3 }
payline line04 { 1,2,3,2,1 }
payline line05 { 3,2,1,2,3 }

payline line01, line02, line03, line04, line05; // instantiate the paylines into the game
```

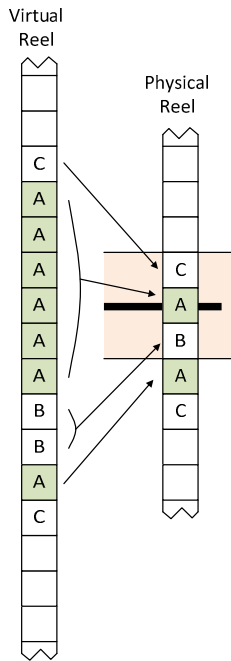
VIRTUAL REELS

Virtual reels were originally used in mechanical stepper games to implement symbol weightings, as described in the Telnaes patent (US 4448419). While not common, some game designers also like to use them in video slots.

In a mechanical stepper machine the reel is a physical device, and mechanical constraints on the size of the reel and symbols limit the number of symbols around the reel to about 22. If there were 100 symbols per reel for example, then each symbol would be so small the player wouldn’t be able to see them, or conversely the gaming machine would be enormous to fit the massive reels.

Considering a 3 reel game this limits the cycle to 22^3 , or 10648 different positions. If we wanted a 20K credit prize, and only one combination of the 10648 games hit, we would still have an RTP from that one prize of around 188%. This severely limits the maximum prizes on the pay table, and flexibility in designing the game.

Virtual reels model a longer virtual reel, which maps multiple virtual stopping positions to a single physical stop position on the actual reel.



In this example we have a virtual reel with one stack of $6 \times A$ symbols and another $1 \times A$ symbol. The virtual symbols map to the physical reel, so that when the virtual reel is spun and a stop position is determined, it determines a corresponding stop position on the physical reel.

When we spin the virtual reel we can see that the chance of a hit on the stack of 6 A symbols is 6 size times higher than hitting the single A symbol, and also 3 times that of hitting the B symbol. Hence we can see that the weighting of the symbols on the virtual reel affects the probability of stopping on a symbol on physical reel.

Consider again the previous example of a 3 reel game, with a single combination that wins 20K credits. We could easily make a virtual reel of 100 symbols, giving a cycle of $100^3 = 1$ million games. The RTP due to that prize is now only 0.2% instead of the 188% we had without the virtual reel.

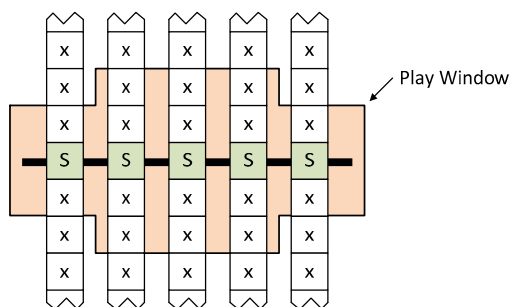
On stepper machines virtual reels allow vastly increased prizes, and flexibility in pay table design.

Unlike normal games, the win (hence RTP) of each combination is different for each payline, greatly complicating Excel spread sheets. To understand why this happens consider a virtual reel with a very large weighting at one position. When the reel spins stops, it is highly likely that this symbol will stop on payline 1, and correspondingly highly unlikely that it will stop on the position above or below that (i.e. on any other payline).

IRREGULAR WINDOW

Most games have the usual 3×3 or 5×3 window, but some games have more unusual shapes. The RTP for these games is easily calculated using the same principals outlined so far. The normal payline combinations are independent of the height of the window; it is only scatter wins that are sensitive to window height.

This example shows a window with 3,5,5,5,3 symbols visible on reels 1 to 5 respectively.



For example the number of hits for a scatter symbol on all 5 reels is 1125.

$$\begin{aligned} hits(S, S, S, S, S) &= 3 \times 5 \times 5 \times 5 \times 3 \\ &= 1125 \end{aligned}$$

In Slot Designer the window is described with the following code, which first defines the maximum size of the window, then defines which locations within it are used.

```
window 5,3,{2,3,4},{1,2,3,4,5},{1,2,3,4,5},{1,2,3,4,5},{2,3,4};
```

PLAYER CHOICE

A feature of some games is to present the player with a set of choices (e.g. boxes) with hidden prizes and ask them to pick one or more of them. As each prize is chosen the value of the prize is revealed, and the player wins the sum of the prizes chosen. These are also known as pick games.

For this simple case the average win is

$$\text{average win} = \text{number of choices} \times \text{average prize value} \quad \text{Equation 23}$$

There are many variations of these types of features, such as picking other feature games, multipliers, or prizes that immediately terminate the choices. Some are quite complex, and to determine the win and are beyond the current scope of this document.

FREE GAMES

A common game feature is free spins, where a base game triggers into a series of free games.

Consider first the case where free games do not retrigger additional free games. The average win of the base game and free game is B_0 and B_1 respectively, and the base game triggers n_1 free games with a trigger probability of p_1 per spin. Hence the average number of free games is $n_1 p_1$

$$E(\text{free games}) = n_1 p_1 \quad \text{Equation 24}$$

The final win, F , of the combined series of base and free games is then

$$F = B_0 + n_1 p_1 B_1 \quad \text{Equation 25}$$

As usual the RTP is the win divided by the bet.

$$\text{final rtp} = \frac{B_0 + n_1 p_1 B_1}{\text{bet}} \times 100\% \quad \text{Equation 26}$$

Now consider what happens when the free game is able to retrigger n_2 games with probability p_2 . The average number of free spins is

$$\begin{aligned} E(\text{freespins}) &= n_1 p_1 + n_1 p_1 (n_2 p_2) + n_1 p_1 (n_2 p_2)^2 + n_1 p_1 (n_2 p_2)^3 + \dots \\ &= n_1 p_1 \sum_{i=1}^{\infty} (n_2 p_2)^i \\ &= n_1 p_1 \left(\frac{1}{1 - n_2 p_2} \right) \end{aligned} \quad \text{Equation 27}$$

And the RTP is again the base win plus the average free game win times the average number of free games, divided by the bet.

$$F = \frac{B_0 + B_1 n_1 p_1 \left(\frac{1}{1 - n_2 p_2} \right)}{\text{bet}} \times 100\% \quad \text{Equation 28}$$

STANDARD DEVIATION

The standard deviation σ is calculated (Equation 29) using the probabilities of each prize in the base and feature games, times the difference between that prizes credit value and the mean credit win of the entire series of base/feature games.

$$\sigma = \sqrt{\sum_{i=1}^n p_i (x_i - \bar{x})^2} \quad \text{Equation 29}$$

The standard deviation suitable for regulator approval can be calculated in Excel. The probability of each prize in a feature game is the total probability of reaching that feature game times the probability of that prize being awarded within the feature game.

The probability of the base game itself is 1.0, and the probability of the free game is the expected value of the free spins as calculated by Equation 27.

While there are usually no zero credit prizes, there is a probability of not winning any credit prize, and this value can be omitted.

VOLATILITY INDEX AND CONFIDENCE INTERVALS

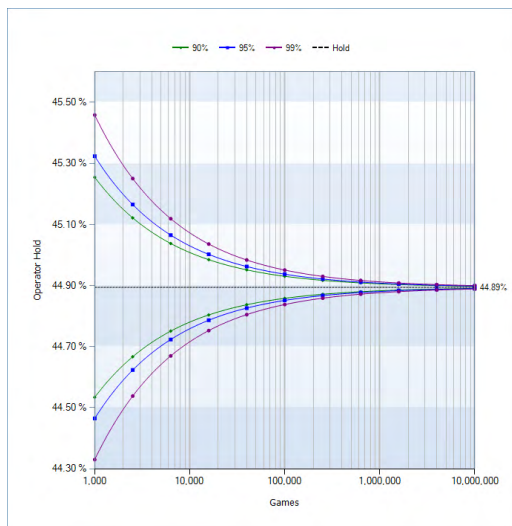
The confidence intervals show the range of the RTP on a gaming machine after N games

$$rtp\ range = rtp \pm \frac{volatility\ index}{\sqrt{N}} \quad \text{Equation 30}$$

Where the volatility index based on a 95% confidence interval is

$$volatility\ index = 1.96 \cdot \sigma \quad \text{Equation 31}$$

If the measured RTP is outside this range then, with 95% confidence, we can say it is not working as expected. This calculation assumes the game has a normal distribution, which may not be the case.



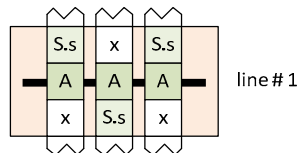
Slot Designer displays the confidence levels to show the expected margin to the game operator, to 90, 95, and 99% confidence levels.

The volatility index based on 90 and 99% confidence intervals is 1.64σ and 2.58σ respectively.

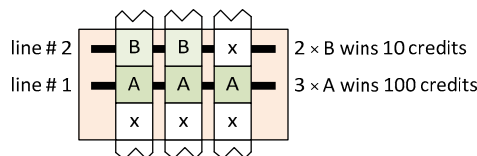
CHAPTER 3 SIMULATION VS THEORETICAL CALCULATION

The calculation of RTP in Excel is a theoretical calculation based on the probability of combinations, and produces statistics for each combination or win independently. We cannot determine the probability of wins occurring on multiple paylines at the same time, or a payline win combined with an off-payline scatter win.

For example, we can calculate the probability of a win due to $3 \times A$, and $3 \times$ scattered S independently, but not the probability of getting both at the same time.



Similarly we can calculate the probability of winning 100 credits and 10 credits independently, but not the probability of winning 110 credits.



A win that comprises multiple independent wins is referred to as a coinciding win, and provides important statistics related to game performance. It gives a more accurate picture of the game from the players perspective, and in some cases is required for regulatory compliance.

When a player wins on multiple paylines their experience is of winning the coinciding win, not the individual parts – i.e. they think they won 110 credits, not 100 and 10.

Coinciding statistics can be determined by creating a simulation of the game with a custom program, for example in C# or C++, so that for each reel stopping position in the game the coinciding win is determined and recorded. This not only provides the coinciding statistics, but can be used for checking the calculations in the spreadsheet before it is implemented on the gaming platform.

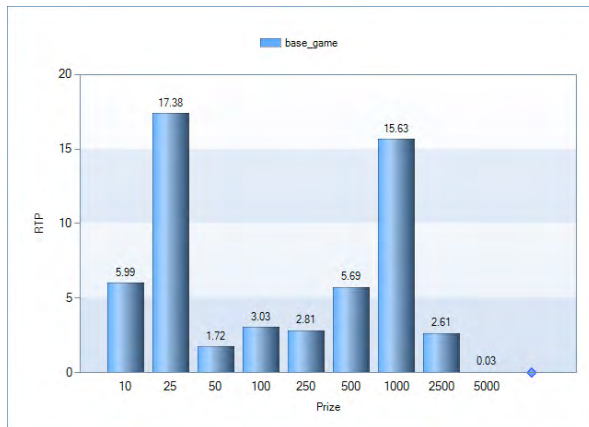
The simulation is typically done in two ways, either as an exhaustive cycle though all possible stop positions (referred to as full cycle), or using a random sampling of the possible outcomes (Monte Carlo). The full cycle simulation allows an exact match to the calculations in the spreadsheet, but is not capable of calculating coinciding statistics across a series of feature games. Monte Carlo simulation calculates coinciding wins across series of base/feature games, but does not provide a perfectly accurate result.

Slot Designer supports all these types of calculations and simulations.

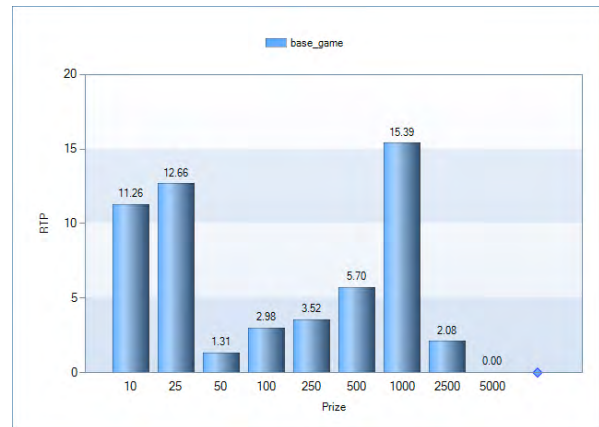
PROFILE

When we look at the profile of the game, dividing all the possible wins into ranges, we get a different picture of the game depending on the use of coinciding or non-coinciding statistics, and while it's easy in Excel to calculate the non-coinciding profile and produce such a chart, it is the coinciding data that is actually more useful.

The following charts show an example of difference between the results depending on how they are determined (these are not from a real game).



Coinciding



Non-coinciding

STANDARD DEVIATION

The standard deviation of the game reduces as we increase the number of paylines (its less volatile), but using Excel we have no way of properly calculating the coinciding standard deviation. Fortunately it is not required for regulatory compliance.

MAXIMUM WINS

In some markets the maximum allowable win of the game is specified by government regulation.

The maximum usually results from a number of separate coinciding payline and scatter wins. For example the win may be limited to \$10000, and while there is no individual win exceeding that amount we also need to ensure there is no possible combination of multiline wins and scatters wins over \$10000 either. We probably also want to have a top pay table prize as close as possible to the limit as its more appealing to the player, yet the closer we get to this prize for a single win the more likely we are to exceed it due to coinciding wins. This can involve numerous iterations of the game as we try variations of the pay table and reelstrip layout to optimize the game.

SIMULATION AND VERIFICATION OF PRODUCT CODE

The implementation of the game in the product code is simulated to provide a final verification of the theoretical game design, and to check for software implementation errors. This type of simulation is generally much slower than any custom simulator the game designer might use due to the overhead of running platform code which has other requirements than just executing the simulation. None the less it is essential to at least check for software bugs in the product.

The game is simulated to model a large number plays and the RTP determined. The RTP should be close to the theoretical RTP calculated by the game designer. While it won't be exactly the same, as the test is a random simulation, a significant discrepancy outside the allowable margin indicates a problem. At this point a comprehensive breakdown of the simulated results to show the statistics for the various parts of the game is essential for quick diagnosis.

If the only information is that the simulated RTP is 90.2% and the theoretical result is 90.0%, it could take a very long time to work out what is causing the error, especially since modifying the game and rerunning large simulations is very time consuming. To make matters worse this typically occurs when the game is almost finished.

Slot Designer provides a verification tool called "Compare Server Play" to aid in verifying the product code and quickly diagnose errors. The product software is modified to write log files recording reel stopping positions and corresponding credits won, and a series of games is randomly played. The verification utility reads the log file and verifies it against the Slot Designer game model. When a difference between the two implementations is found, the reel stopping positions (essentially the symbols on the screen), and the two different win values are reported. From there is easy diagnose the problem. The full product code simulation should still be run, but by that point an entire class of potential errors has been eliminated.

The calculations are illustrated by calculating several exemplary values from which the spreadsheet can be understood.

First we calculate the hits for the payline combinations and the scatters to determine the average base and free game win. We then determine the probability of the free games occurring, and combine the base and free games wins to calculate the final RTP of the game. Last we calculate the standard deviation.

PAYLINE COMBINATIONS HITS

The calculation of hits for left 4 P1 is shown, and includes the wild symbols.

$$\begin{aligned} \text{hits}(\text{left 4 P1}) &= \text{hits}(P1, P1, P1, P1, \#P1) \\ &= (5 + 1) \times (4 + 1) \times (4 + 1) \times (4 + 2) \times (48 - 4 - 3) \\ &= 36900 \end{aligned} \quad \text{Equation 32}$$

The other non-scatter hits are calculated in a similar manner.

5x3Analysis.xlsx - Microsoft Excel

File Home Insert Page Layout Formulas Data Review View Add-Ins Load Test Acrobat PKZIP Team

fx Insert Function AutoSum Recently Used Financial Logical Text Date & Time Lookup & Reference Math Functions More Name Manager Define Name Use in Formula Trace Precedents Trace Dependents Remove Arrows Show Formula Error Checking Evaluate Formula

Function Library Defined Names

C28 $= (B11+B\$9) * (C11+C\$9) * (D11+D\$9) * (E11+E\$9) * (F\$21-(F11+F\$9))$

1 Pays left to right except scatter which pays any
2 5,4,3 scatters pay 25, 15, 10 free games
3 Free games are the same as the base game
4 Wild substitutes for all symbols except scatter
5 Bet 1
6
7 Symbol Distribution

	Reel 1	Reel 2	Reel 3	Reel 4	Reel 5
8 wild	1	1	1	2	3
9 scat	3	3	3	2	1
10 P1	5	4	4	4	4
11 P2	6	5	4	5	4
12 P3	6	4	5	4	5
13 P4	5	3	6	3	5
14 A	10	11	12	11	10
15 K	9	8	9	8	5
16 Q	8	6	8	6	4
17 J	7	8	9	8	3
18 ten	6	7	8	9	2
19 nine	10	9	8	7	2
20 Total	76	69	77	69	48
21 Cycle	1337345856				
22 Hits					
23	5	4	3	2	1
24 Total					
25 scat	13122	618921	11366892		11998935
26 P1	6300	36900	453600		496800
27 P2	10290	60270	624960		695520
28 P3	10080	50400	635040		695520
29 P4	6720	33600	516096		556416
30 A	290004	780780	4612608		5683392
31 K	72000	360000	2548800		2980800
32 Q	31752	185976	1660176		1877904

hits(left 4 P1) = 36900

Hits Any 4 scat

scat, scat, scat, scat, #scat
scat, scat, scat, #scat, scat
scat, scat, #scat, scat, scat
scat, #scat, scat, scat, scat
#scat, scat, scat, scat, scat
Total

Hits Any 3 scat

scat, scat, scat, #scat, #scat
scat, scat, #scat, scat, #scat
scat, scat, #scat, scat, scat
scat, #scat, scat, scat, #scat
#scat, scat, scat, scat, scat
#scat, #scat, scat, scat, scat
#scat, scat, scat, scat, scat
#scat, scat, scat, scat, scat
#scat, #scat, scat, scat, scat
Total

Probability

5	4	
scat	9.81197E-06	0.000462798
P1	4.71082E-06	2.7592E-05
P2	7.69434E-06	4.50689E-05

SCATTER COMBINATION HITS

Scatter hits are a little more complex as they are “any” pay rules rather than left to right, and are the sum of the different combinations that make up the rule. For the rule “any 4 scat” we need to calculate the hits for 5 combinations, the first of which is

$$\begin{aligned} hits(scatter, scatter, scatter, scatter, \#scatter) &= (3 * 3) \times (3 * 3) \times (3 * 3) \times (2 * 3) \times (48 - 1 * 3) \\ &= 196830 \end{aligned} \quad \text{Equation 33}$$

hits(scatter,scatter,scatter,scatter,#scatter)=196830

Symbol	Reel 1	Reel 2	Reel 3	Reel 4	Reel 5
wild	1	1	1	2	3
scat	3	3	3	2	1
P1	5	4	4	4	4
P2	6	5	4	5	4
P3	6	4	5	4	5
P4	5	3	6	3	5
A	10	11	12	11	10
K	9	8	9	8	5
Q	8	6	8	6	4
J	7	8	9	8	3
ten	6	7	8	9	2
nine	10	9	8	7	2
Total	76	69	77	69	48

	5	4	3	2	1	Total
scat	13122	618921	11366892			11998935
P1	6300	36900	453600			496800
P2	10290	60270	624960			695520
P3	10080	50400	635040			695520
P4	6720	33600	516096			556416
A	290004	780780	4612608			5683392
K	72000	360000	2548800			2980800
Q	31752	185976	1660176			1877904
J	43200	302400	2039040			2384640

Hits Any 4 scat	
scat,scat,scat,scat,#scat	196830
scat,scat,scat,#scat,scat	137781
scat,scat,#scat,scat,scat	99144
scat,#scat,scat,scat,scat	87480
#scat,scat,scat,scat,scat	97686
Total	618921

AVERAGE WIN

From the hits we can calculate the probability of each rule, and from that the average credit win.

The probability of “left 4 P1” is 2.7592×10^{-5} .

$$\begin{aligned} probability(left\ 4\ p1) &= \frac{hits(left\ 4\ p1)}{cycle} \\ &= \frac{36900}{1337345856} \\ &= 2.7592 \cdot 10^{-5} \end{aligned} \quad \text{Equation 34}$$

The average win for “left 4 P1” is then 0.020694 credits.

$$\begin{aligned} win(left\ 4\ p1) &= probability \times prize \\ &= 2.7592 \cdot 10^{-5} \times 750 \\ &= 0.020694 \end{aligned} \quad \text{Equation 35}$$

The total win of the base game is the sum of the wins of all the rules in the pay table. As the free game has identical rules to the base game it has the same average win.

16 K 19 9 9 8 5
 17 Q 8 6 8 4
 18 J 7 8 9 3
 19 ten 6 7 8 2
 20 nine 10 9 8 2
 21 Total 76 69 77 69 48
 22
 23 Cycle 133734586 probability(left 4 p1)= hits/cycle
 24 = 36900/133734586
 25 = 2.7592E-05 credits
 26 Hits 5 4 3 2 1 Total
 27 scat 13122 618921 11366892 11998935
 28 P1 6300 36900 453600 496800
 29 P2 10290 60270 624960 695520
 30 P3 10080 50400 635040 695520
 31 P4 6720 33600 516096 956416
 32 A 290004 780780 4612608 5683392
 33 K 72000 360000 2548800 2980800
 34 Q 31752 185976 1660176 1877904
 35 J 43200 302400 2039040 2384640
 36 ten 27720 238392 1403136 12612096 14281344
 37 nine 44550 383130 2851200 24773760 28052640
 38 Total 542616 2431848 17344056 37385856 0 57704976
 39
 40 Pay Table 5 4 3 2 1
 41
 42 scat 100 20 10
 43 P1 1000 750 500
 44 P2 750 500 100
 45 P3 750 100 15
 46 P4 500 40 15
 47 A 400 50 15
 48 K 400 50 10
 49 Q 200 25 10
 50 J 200 25 10
 51 ten 150 25 10 5
 52 nine 150 25 10 5
 53
 54 Win
 55
 56 5 4 3 2 1 Total
 57 scat 0.000981 0.006256 0.084996 0.000000 0.000000 0.095233
 58 P1 0.004111 0.020694 0.169590 0.000000 0.000000 0.194994
 59 P2 0.005771 0.022333 0.046731 0.000000 0.000000 0.075036
 60 P3 0.005653 0.003769 0.007123 0.000000 0.000000 0.016544
 61 P4 0.002512 0.001005 0.005789 0.000000 0.000000 0.009306
 62 A 0.086740 0.029191 0.051736 0.000000 0.000000 0.167668
 63 K 0.021535 0.013459 0.019059 0.000000 0.000000 0.054053
 64 Q 0.004749 0.003477 0.012144 0.000000 0.000000 0.020639
 65 J 0.006461 0.005653 0.015247 0.000000 0.000000 0.027360
 66 ten 0.000981 0.006256 0.084996 0.000000 0.000000 0.095233
 67 nine 0.004111 0.020694 0.169590 0.000000 0.000000 0.194994
 68 Total 0.000981 0.006256 0.084996 0.000000 0.000000 0.095233
 69
 70 Hits Any 3 scat 2066715
 71 scat, scat, scat, scat, scat 1487160
 72 scat, scat, scat, scat, scat 1041012
 73 scat, scat, scat, scat, scat 1312200
 74 scat, scat, scat, scat, scat 918540
 75 scat, scat, scat, scat, scat 660960
 76 scat, scat, scat, scat, scat 1465290
 77 scat, scat, scat, scat, scat 1025703
 78 scat, scat, scat, scat, scat 738072
 79 scat, scat, scat, scat, scat 651240
 80 Total 11366892
 81
 82 Probability 5 4 3 2 1
 83 scat 9.84197E-06 5.000462758 0.00849959 0 0
 84 P1 4.71082E-06 2.7592E-05 0.000339179 0 0
 85 P2 7.69434E-06 4.50669E-05 0.000467314 0 0
 86 P3 7.57332E-06 3.76866E-05 0.000474851 0 0
 87 P4 5.02488E-06 2.51244E-05 0.000385911 0 0
 88 A 0.00021685 0.000583828 0.003449076 0 0
 89 K 5.3838E-05 0.00026919 0.001905805 0 0
 90 Q 2.37425E-05 0.000139064 0.001241396 0 0
 91 J 3.23028E-05 0.00022612 0.001524692 0 0
 92 ten 2.07276E-05 0.000178258 0.001049195 0.009430651 0
 93 nine 3.33123E-05 0.000286485 0.002131984 0.018524572 0
 94
 95 Base Game p(win-ave-win)^2 5 4 3 2 1
 96 scat 0.09628631 0.168150268 0.697880753 0 0
 97 P1 4.701983554 15.48165469 84.47673822 0 0
 98 P2 4.31741917 11.2244555 45.85817139 0 0
 99 P3 4.229134939 0.369823963 0.093887919 0 0
 100 P4 1.251507233 0.038334468 0.076302563 0 0
 101 A 34.53341501 1.405281907 0.681954157 0 0
 102 K 8.573695125 0.647943706 0.156485912 0 0
 103 Q 0.94808026 0.080510435 0.101928027 0 0
 104 J 1.280011239 0.130911276 0.12518873 0 0
 105 ten 0.460552806 0.103201723 0.086146821 0.155553073 0
 106 nine 0.740174153 0.165859912 0.175052037 0.305550679 0
 107
 108 Free Game p(win-ave-win)^2 5 4 3 2 1
 109 scat 0.009777278 0.017074618 0.070865466 0 0
 110 P1 0.477457294 1.572066101 8.578089306 0 0
 111 P2 0.438389165 1.139773903 0.465661314 0 0

COMBINE BASE AND FREE GAME WIN TO CALCULATE FINAL RTP

Using Equation 28 (page 14) we calculate the final RTP. First, the expected number of free spins triggered, per spin, by the base game is the sum of the individual triggers, 0.092183173

$$\begin{aligned} freespins &= \sum n \cdot p \\ &= 25 \times 9.81197 \cdot 10^{-6} + 15 \times 0.000462798 + 10 \times 0.00849959 \\ &= 0.092183173 \end{aligned} \tag{Equation 36}$$

The number of free spins for the free game is the same as the base game, as the games are identical (except for the bet), and the free game retriggers.

The final RTP is then 93.867699%

$$F = \frac{B_0 + B_1 n_1 p_1 \left(\frac{1}{1 - n_2 p_2} \right)}{bet} \times 100\% \quad \text{Equation 37}$$

$$= \frac{0.852147 + 0.852147 \times 0.092183173 \left(\frac{1}{1 - 0.092183173} \right)}{1} \times 100\%$$

$$= 93.867699\%$$

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
64	Q	0.004749	0.003477	0.012414	0.000000	0.000000	0.020639		P1	0.477457294	1.572066101	8.578089306	0	0
65	J	0.006461	0.005653	0.015247	0.000000	0.000000	0.027360		P2	0.438389105	1.139773903	0.465661314	0	0
66	ten	0.003109	0.004456	0.010492	0.007153	0.000000	0.065211		P3	0.429442447	0.037553332	0.009533736	0	0
67	nine	0.004997	0.007162	0.021320	0.092623	0.000000	0.126102		P4	0.127082805	0.003892628	0.007748052	0	0
68	Total	0.147219	0.120656	0.444496	0.139476	0.000000	0.852147		A	3.506654308	0.14269767	0.069248219	0	0
69									K	0.870605613	0.065794668	0.015890175	0	0
70	Base Game Win			0.852147					Q	0.095533249	0.008175336	0.01035016	0	0
71	Feature Game Win			0.852147					J	0.12997721	0.013293229	0.01271214	0	0
72									ten	0.046766284	0.010479495	0.008747676	0.015795451	0
73	Trigger N scat	5	4	3					nine	0.075160099	0.016842046	0.01777545	0.031026778	0
74	Free Spins	25	15	10					Total	201.2073571				
75	Probability	9.81197E-06	0.000462798	0.00849959					Std.Dev	14.18475792				
76	Freespins	0.000245299	0.00694197	0.084995904										
77														
78	Free spins			0.092183173										
79	Retrigger free spins			0.092183173										
80	E(freespins)			0.10154380										
81														
82														
83	Final Win			0.93867699										
84	Final RTP			93.867699%										
85														
86														
87														
88														
89														
90														
91														
92														
93														
94														
95														
96														
97														

STANDARD DEVIATION

We calculate the standard deviation using Equation 29 (page 14). We build two tables for containing the probability of a prize occurring times the square of the prize minus the games final win. Each cell contains

$$p_i(x_i - \bar{x})^2 \quad \text{Equation 38}$$

The probability for base game prizes is simply the probability of the combination occurring, which we calculated as 2.7592×10^{-5} for left 4 P1 with Equation 34 (page 20). Thus the cell representing left 4 P1 contains

$$2.7592 \cdot 10^{-5} (750 - 0.93867699)^2 = 15.48165469 \quad \text{Equation 39}$$

The corresponding cell in the free game uses takes into account the probability of free game occurring (after the initial trigger and any number of retriggers), which is the expected number of free spins from Equation 27 (page 14), and as shown in the similar Equation 37, is 0.10154380.

$$\begin{aligned} E(\text{freespins}) &= n_1 p_1 \left(\frac{1}{1 - n_2 p_2} \right) \\ &= 0.852147 \times 0.092183173 \left(\frac{1}{1 - 0.092183173} \right) \\ &= 0.10154380 \end{aligned} \quad \text{Equation 40}$$

The table entry for left 4 P1 in the free game is then 1.572066101

$$0.10154380 \times 15.48165469 = 1.572066101 \quad \text{Equation 41}$$

The standard deviation is then the square root of the sum of these two tables

$$\sigma = \sqrt{201.2073571}$$

$$= 14.18475792$$

Equation 42

The relevant part of the spread sheet in more detail

30	P3	10080	50400	635040			695520	Probability					
31	P4	6720	33600	516096			556416		5	4	3	2	1
32	A	290004	780780	4612608			5683392	scat	9.81197E-06	0.000462798	0.00849959	0	0
33	K	72000	360000	2548800			2980800	P1	4.71082E-06	2.7592E-05	0.000339179	0	0
34	Q	31752	185976	1660176			1877904	P2	7.69431E-06	4.50669E-05	0.000467314	0	0
35	J	43200	302400	2039040			2384640	P3	7.53732E-06	3.76866E-05	0.000474851	0	0
36	ten	27720	238392	1403136	12612096		14281344	P4	5.02488E-06	2.51244E-05	0.000385911	0	0
37	nine	44550	383130	2851200	24773760		28052640	A	0.00021685	0.000583828	0.003449076	0	0
38	Total	542616	2431848	17344656	37385856	0	57704976	K	5.3838E-05	0.00026919	0.001905865	0	0
39								Q	2.37425E-05	0.000139064	0.001241396	0	0
40	Pay Table							J	3.23028E-05	0.00022612	0.001524692	0	0
41		5	4	3	2	1		ten	2.07276E-05	0.000178258	0.001049195	0.009430691	0
42	scat	100	20	10				nine	3.33128E-05	0.000286485	0.002131984	0.018524572	0
43	P1	1000	750	500									
44	P2	750	500	100									
45	P3	750	100	15									
46	P4	500	40	15									
47	A	400	50	15									
48	K	400	50	10									
49	Q	200	25	10									
50	J	200	25	10									
51	ten	150	25	10	5								
52	nine	150	25	10	5								
53													
54													
55	Win												
56		5	4	3	2	1	Total						
57	scat	0.000981	0.009256	0.084996	0.000000	0.000000	0.095233						
58	P1	0.004711	0.020694	0.169590	0.000000	0.000000	0.194994						
59	P2	0.005771	0.022533	0.046731	0.000000	0.000000	0.075036						
60	P3	0.005653	0.003769	0.007123	0.000000	0.000000	0.016544						
61	P4	0.002512	0.001005	0.005789	0.000000	0.000000	0.009306						
62	A	0.086740	0.029191	0.051736	0.000000	0.000000	0.167668						
63	K	0.021535	0.013459	0.019059	0.000000	0.000000	0.054053						
64	Q	0.004749	0.003477	0.012414	0.000000	0.000000	0.020639						
65	J	0.006461	0.005653	0.015247	0.000000	0.000000	0.027360						
66	ten	0.003109	0.004456	0.010492	0.047153	0.000000	0.065211						
67	nine	0.004997	0.007162	0.021320	0.092623	0.000000	0.126102						
68	Total	0.147219	0.120656	0.444496	0.139776	0.000000	0.852147						
69													
70	Base Game Win		0.852147										
71	Feature Game Win		0.852147										
72													
73	Trigger N scat	5	4	3									
74	Free Spins	25	15	10									
75	Probability	9.81197E-06	0.000462798	0.00849959									
76	Free spins	0.000245299	0.00694197	0.084995904									
77													
78	Free spins			0.092183173									
79	Retrigger free spins			0.092183173									
80	E(freespins)			0.10154380									
81													
82													
83	Final Win		0.93867699										
84	Final RTP		93.867699%										
85													
86													
87													

ANALYSIS USING SLOT DESIGNER

The same is shown in the Slot Designer IDE. Unlike in Excel where the calculations are specified, Slot Designer performs whatever calculations are required itself. The 'code' describing the game shown on the right, and the calculated game statistics on the left.

The screenshot displays the Slot Designer IDE interface. The left pane shows the 'Score [Combination]' table, which lists various game outcomes, their hit rates, probabilities, RTP percentages, prize values, and rules. The right pane shows the '5x3FreeGame.sd' code, which defines the game's logic, including symbol wilds, reels, and payouts.

Score [Combination] Table:

Score [Combination]	Hits	Hit Rate	Probability	RTP %	Prize	Prize Value	Rule
69,703,911	19	0.000047108	0.4710823286	1000	1000	left 5 P1	
36,900	36,242	0.0000275920	2.0693973721	750	750	left 4 P1	
453,600	2,948	0.0003391793	16.9589638299	500	500	left 3 P1	
10,290	129,966	0.0000076943	0.5770758525	750	750	left 5 P2	
60,270	22,189	0.0000450669	2.2533438052	500	500	left 4 P2	
624,960	2,140	0.0004673137	4.6731366998	100	100	left 3 P2	
10,080	132,673	0.0000075373	0.5652987943	750	750	left 5 P3	
50,400	26,535	0.0000376866	0.3768658629	100	100	left 4 P3	
635,040	2,106	0.0004748510	0.7122746809	15	15	left 3 P3	
6,720	199,010	0.0000050249	0.2512439086	500	500	left 5 P4	
33,600	39,802	0.0000251244	0.1004975634	40	40	left 4 P4	
516,086	2,591	0.0003859106	0.5788659654	15	15	left 3 P4	
290,004	4,611	0.0002168504	8.6740164842	400	400	left 5 A	
780,780	1,713	0.0005838280	2.9191401629	50	50	left 4 A	
4,612,608	290	0.003490764	5.1736145657	15	15	left 3 A	
72,000	18,574	0.0000538380	2.1535192165	400	400	left 5 K	
360,000	3,715	0.0002691899	1.3459495103	50	50	left 4 K	
2,548,800	525	0.0019058645	1.9058645066	10	10	left 3 K	
31,752	42,118	0.0000237425	0.4748509872	200	200	left 5 Q	
185,976	7,191	0.0001390635	0.3476587585	25	25	left 4 Q	
1,660,176	806	0.0012413962	1.2413961524	10	10	left 3 Q	
43,200	30,957	0.0000323028	0.460557649	200	200	left 5 J	
302,400	4,422	0.0002261195	0.5652987943	25	25	left 4 J	
2,039,040	656	0.0015246916	1.5246916053	10	10	left 3 J	
27,720	48,245	0.0000207276	0.3109143369	150	150	left 5 ten	
238,392	5,610	0.0001782576	0.4456438829	25	25	left 4 ten	
1,403,136	953	0.0010491946	1.0491945623	10	10	left 3 ten	
12,612,096	106	0.0094306914	4.7153456764	5	5	left 2 ten	
44,500	30,019	0.0000333123	0.4996837557	150	150	left 5 nine	
383,130	3,491	0.0002864854	0.7162133832	25	25	left 4 nine	
2,851,200	469	0.0021319840	2.1319840243	10	10	left 3 nine	
24,773,750	54	0.0185245723	9.2622861502	5	5	left 2 nine	
13,122	101,916	0.0000098120	0.0981197193	100	100	any 5 scat @ scatter_line	
618,921	2,161	0.0004627980	0.9255960187	20	20	any 4 scat @ scatter_line	
11,366,892	118	0.0084995904	8.4995904006	10	10	any 3 scat @ scatter_line	
11,998,935	111	0.0084995904	8.6530224043	10	10	any 3 scat @ scatter_line	
13,122	101,916	0.0000098120	0.0230256808	repeat(free_game,25)	any 5 scat @ scatter_line		
618,921	2,161	0.0004627980	0.6516267656	repeat(free_game,15)	any 4 scat @ scatter_line		
11,366,892	118	0.0084995904	7.9783699579	repeat(free_game,10)	any 3 scat @ scatter_line		

Game Summary [Combination] Table:

Game	Bet	Ave. Bet	Base RTP	Base Win	Feature RTP	Feature Win	Total RTP	Std Dev Final	Total Win
base_game	1	1.000	85.214677	0.852147	8.653022	0.086530	93.867699	14.184758	0.93867
free_game	0	0.000		0.852147		0.086530			0.93867

Symbol Distribution [strips] Table:

	Reel 1	Reel 2	Reel 3	Reel 4	Reel 5
wild	1	1	1	2	3
scat	3	3	3	2	1
P1	5	4	4	4	4
P2	6	5	4	5	4
P3	6	4	5	4	5
P4	5	3	6	3	5
A	10	11	12	11	10
K	9	8	9	8	5
Q	8	6	8	6	4
J	7	8	9	8	3
ten	6	7	8	9	2
nine	10	9	8	7	2
Total	76	69	77	69	48

Code (5x3FreeGame.sd):

```

1 symbol wild, scat, P1, P2, P3, P4, A, K, Q, J, ten, nine;
2
3 window 5,3;
4
5 base ( base_game )
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```

The Slot Designer code for the game.

```
symbol wild, scat, P1, P2, P3, P4, A, K, Q, J, ten, nine;

window 5,3;

base { base_game }

payline line01 { 2,2,2,2,2 }

substitute wild = all except scat;

payline line01;

scatter scat;

pay 1000,750,500 on left 5,4,3 p1;
pay 750,500,100 on left 5,4,3 p2;
pay 750,100,15 on left 5,4,3 p3;
pay 500,40,15 on left 5,4,3 p4;
pay 400,50,15 on left 5,4,3 A;
pay 400,50,10 on left 5,4,3 K;
pay 200,25,10 on left 5,4,3 Q;
pay 200,25,10 on left 5,4,3 J;
pay 150,25,10,5 on left 5,4,3,2 ten;
pay 150,25,10,5 on left 5,4,3,2 nine;
pay 100,20,10 on any 5,4,3 scat;

reelstrips strips;

pay repeat(free_game,25),repeat(free_game,15),repeat(free_game, 10) on any 5,4,3 scat;

game base_game
{
}

game free_game
{
    bet = 0;
}

reelstrips strips
{
    reel 1:
    {
        wild,P2,scat,P3,P4,A,K,Q,A,
        J,J,ten,nine,P1,P2,P3,A,A,K,
        Q,P4,J,ten,nine,A,scat,P1,P2,P3,
        K,K,Q,A,J,nine,P4,ten,K,nine,
        P1,P2,P3,A,K,Q,J,K,nine,nine,
        ten,A,scat,P1,P2,P3,P4,K,Q,A,
        Q,J,nine,ten,nine,nine,P1,P2,P3,A,
        K,P4,Q,Q,J,nine,ten
    },
    reel 2:
    {
        wild,P2,scat,P3,P4,A,A,K,Q,
        J,ten,nine,A,K,nine,P1,P2,K,A,
        J,P3,K,Q,scat,ten,A,Q,P4,J,
        P1,P2,A,K,J,ten,nine,nine,A,P3,
        Q,J,K,J,P1,P2,A,scat,ten,ten,
        nine,P4,A,K,Q,J,P3,nine,P1,P2,
        A,K,ten,ten,J,Q,nine,A,nine,nine
    },
    reel 3:
    {
        wild,P1,P2,scat,P3,P4,A,A,K,Q,
        J,ten,nine,A,K,nine,nine,K,P4,P3,
        A,P1,P2,K,Q,Q,scat,A,J,ten,
        nine,P4,K,A,P3,Q,J,J,nine,ten,
        P1,P2,A,K,P4,Q,J,J,A,ten,
        P3,nine,scat,K,A,Q,J,P4,ten,P1,
        P2,A,K,ten,J,P3,Q,A,ten,nine,
        P4,K,nine,J,A,ten,Q
    },
    reel 4:
    {
        wild,P1,P2,scat,P3,P4,A,K,K,Q,
        J,ten,A,ten,nine,nine,P2,K,A,P1,
        Q,P3,Q,J,A,ten,K,ten,P4,nine,
        P2,A,Q,J,wild,P1,A,scat,P3,K,
        J,ten,J,A,P2,Q,ten,nine,K,A,
        J,P4,J,P1,ten,P3,A,K,P2,Q,
        J,ten,A,ten,nine,K,nine,nine,A
    },
    reel 5:
    {
        wild,P1,P2,scat,P3,P4,A,K,Q,A,
        J,A,ten,P3,P1,P2,wild,P4,A,A,
        K,Q,nine,P3,A,K,P1,P2,P4,A,
        Q,J,wild,P3,A,K,J,P4,P1,P2,
        A,ten,Q,P3,A,K,nine,P4
    }
}
```

In more detail the report shows exactly the same statistics as we calculated in Excel.

5x3FreeGame - Slot Designer 4

File Edit View Tools Help

Analyse Simulate Play Cancel Select All Game base_game

Score [Combination] [Final RTP 93.8676992874]

Hits	Hit Rate	Probability	RTP %	Prize	Prize Value	Rule
69,703,911	19	0.0000047108	85.2146768831	1000	1000	left 5 P1
36,000	36,242	0.0000275920	2.0693973721	750	750	left 4 P1
36,900	6,242	0.0000275920	2.0693973721	750	750	{*,*,*,*,{wild,P1},{wild,P1},{wild,P1},{wild,P1},{wild,P1},*,*,*,*}
453,800	18,948	0.0003391793	16.9589638299	500	500	left 3 P1
10,290	129,966	0.0000076943	0.5770758525	750	750	left 5 P2
50,270	22,189	0.0000456669	2.2533438052	500	500	left 4 P2
624,960	2,140	0.0004673137	4.6731366998	100	100	left 3 P2
10,080	132,673	0.0000075373	0.5639987943	750	750	left 5 P3
50,400	26,535	0.0000376866	0.3768658623	100	100	left 4 P3
635,040	2,106	0.0004748510	0.7122764809	15	15	left 3 P3
6,720	199,010	0.0000050249	0.2512439086	500	500	left 5 P4
33,600	39,802	0.0000251244	0.1004975634	40	40	left 4 P4
516,096	2,591	0.0003859106	0.5788659654	15	15	left 3 P4
290,004	4,611	0.0002168504	8.6740164842	400	400	left 5 A
780,780	1,713	0.00005838280	2.9191401629	50	50	left 4 A
4,612,608	290	0.0034490764	5.1736145657	15	15	left 3 A
72,000	18,574	0.0000538380	2.1535192165	400	400	left 5 K
360,000	3,715	0.0002691899	1.3459495103	50	50	left 4 K
2,548,800	525	0.0019058645	1.9058645066	10	10	left 3 K
31,752	42,118	0.0000237425	0.4748509872	200	200	left 5 Q
185,976	7,191	0.0001390635	0.3476587585	25	25	left 4 Q
1,660,176	806	0.0012413962	1.2413961524	10	10	left 3 Q
43,200	30,957	0.0000323028	0.6460557649	200	200	left 5 J
302,400	4,422	0.0002261195	0.5652987943	25	25	left 4 J
2,039,040	656	0.0015246916	1.5246916053	10	10	left 3 J
27,720	48,245	0.0000207276	0.3109143369	150	150	left 5 ten
238,392	5,610	0.0001782576	0.4456438829	25	25	left 4 ten
1,403,136	953	0.0010491946	1.0491945623	10	10	left 3 ten
12,612,096	106	0.0094306914	4.7153456764	5	5	left 2 ten
44,550	30,019	0.0000333123	0.4996837557	150	150	left 5 nine
383,130	3,491	0.0002864854	0.7162133832	25	25	left 4 nine
2,851,200	469	0.0021319840	2.1319840243	10	10	left 3 nine
24,773,760	54	0.0185245723	9.2622861502	5	5	left 2 nine
13,122	101,916	0.0000089120	0.0981197193	100	100	any 5 scat
618,921	2,161	0.0004627980	0.9255960187	20	20	any 4 scat
196,830	6,794	0.0001471796	0.2943591579	20	20	{*,*,*,*,scat,s,scat,s,scat,s,scat,s,*,*,*,*}
137,781	9,706	0.0001030257	0.2060514105	20	20	{*,*,*,*,scat,s,scat,s,scat,s,scat,s,scat,s,*,*,*,*}
99,144	13,489	0.0000741349	0.1482697981	20	20	{*,*,*,*,scat,s,scat,s,scat,s,scat,s,scat,s,*,*,*,*}
87,480	15,287	0.0000654131	0.1308262924	20	20	{*,*,*,*,scat,s,scat,s,scat,s,scat,s,scat,s,*,*,*,*}
97,686	13,690	0.0000730447	0.1460893598	20	20	{*,*,*,*,scat,s,scat,s,scat,s,scat,s,scat,s,*,*,*,*}
11,366,892	118	0.0084995904	8.4995904006	10	10	any 3 scat
11,998,935	111	0.0000098120	8.6530224043			Coinciding Feature
13,122	101,916	0.0000098120	0.0230256808	repeat(free_game,25)		any 5 scat
618,921	2,161	0.0004627980	0.6516267656	repeat(free_game,15)		any 4 scat
11,366,892	118	0.0084995904	7.9783699579	repeat(free_game,10)		any 3 scat

Hits for left 4 P1

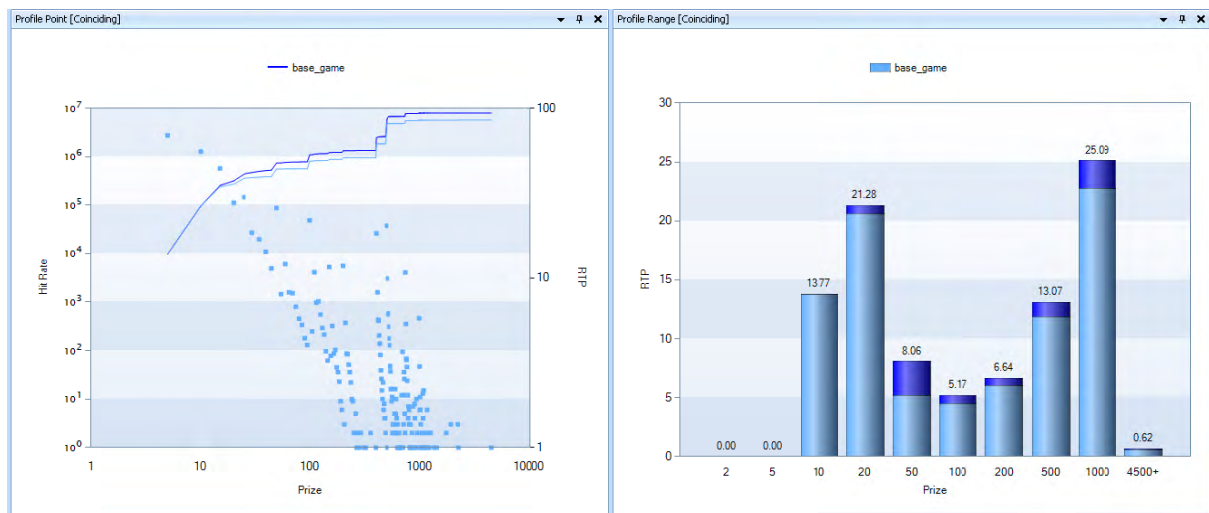
First scatter combination

Standard Deviation

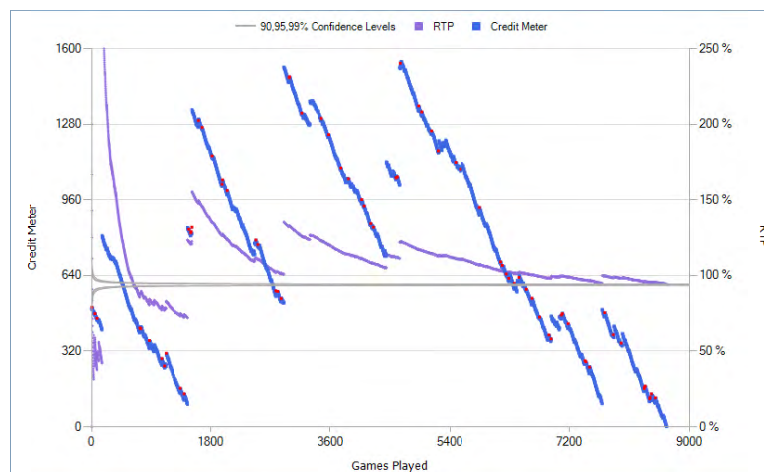
Game Summary [Combination]													
Game	Bet	Ave. Bet	Base RTP	Base Win	Feature RTP	Feature Win	Total RTP	Std Dev Final	Total Win	Hit Rate	Max Win	Std Dev Base	Skewness
base_game	1	1.000	85.214677	0.852147	8.653022	0.086530	93.867699	14.184758	0.938677	19.186095		13.520311	36.251603
free_game	0	0.000		0.852147		0.086530			0.938677	19.186095		13.520311	36.251603

We can also easily examine many other game statistics.

A random simulation (Monte Carlo) simulation of the game, as if it were being played, shows the coinciding wins and how they are split between base and feature games (light/dark blue for base/feature respectively). These charts are from simulating the game for 100M plays over 12 seconds of simulation. This could also be calculated by a custom simulation in C++ or C# (for example), but not Excel.



We can also model play over a number of gaming sessions, and look at each session in detail. This particular session is very close to average, being only 0.03 standard deviations above the sample mean. The blue line indicates the credit meter, and the red dots are the credit meter when free game wins occur.



A playable game is automatically generated from the mathematics enabling the game designer to get an immediate feel for how the game performs. Artwork can easily be added if required to further improve understanding of the game.

